Handout 4.1 **Name:** ____________________
Week # 2: 1/22/2021
Math 1150: Winter 2021
Instructor: Andrade

Introduction:
Mathematics is often used as means to explain or predict. In this exercise, you will be asked to critically analyze a scenario and, equipped with some basic facts, determine whether the prediction is plausible or not.

Having Kittens

This is a poster made by a TNR program, encouraging people to have their cats spayed so they can't have kittens. The activity is about what happens if you don't have your cat spayed. Your task is to decide whether the statement on the poster is correct.

Is it realistic that one female cat would produce 2000 descendants in 18 months?

Question 1. What information might you need to assess whether the statement in the poster is realistic?

Question 2. List some ways which you might be able to tackle this problem.

List some facts which will help you decide:

-
-
-
-
-

Question 3. Given the facts, determine whether you believe the statement is realistic or not. Explain your reasoning- mathematically, if possible.

Group Work

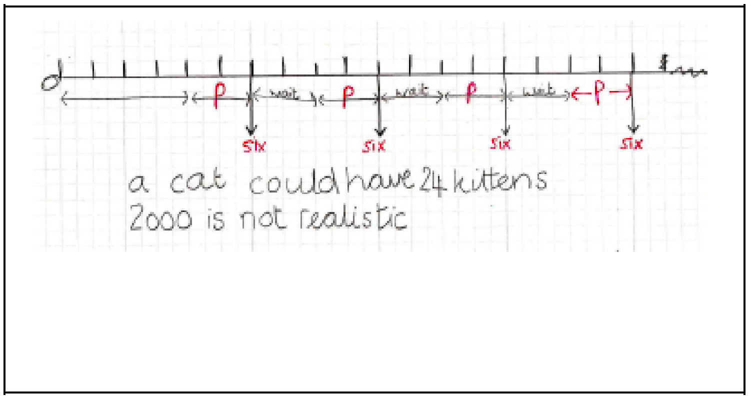
Now you will collaborate with a group to improve your model. Take turns to explain how you did the task and how you think it could be improved. In a moment, you will put your individual work aside and try to produce a joint solution to the problem.

Question 4. Give a brief description of the approaches each member in your group used.

Before attempting to improve your model, take a look at the following sample solutions and consider the following questions:

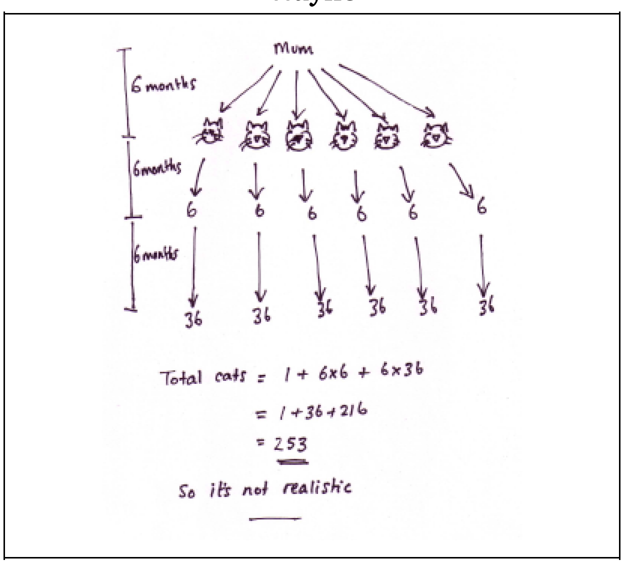
- What have they done correctly?
- What assumptions were made? Were they reasonable?
- How could the solution be improved?

Alice



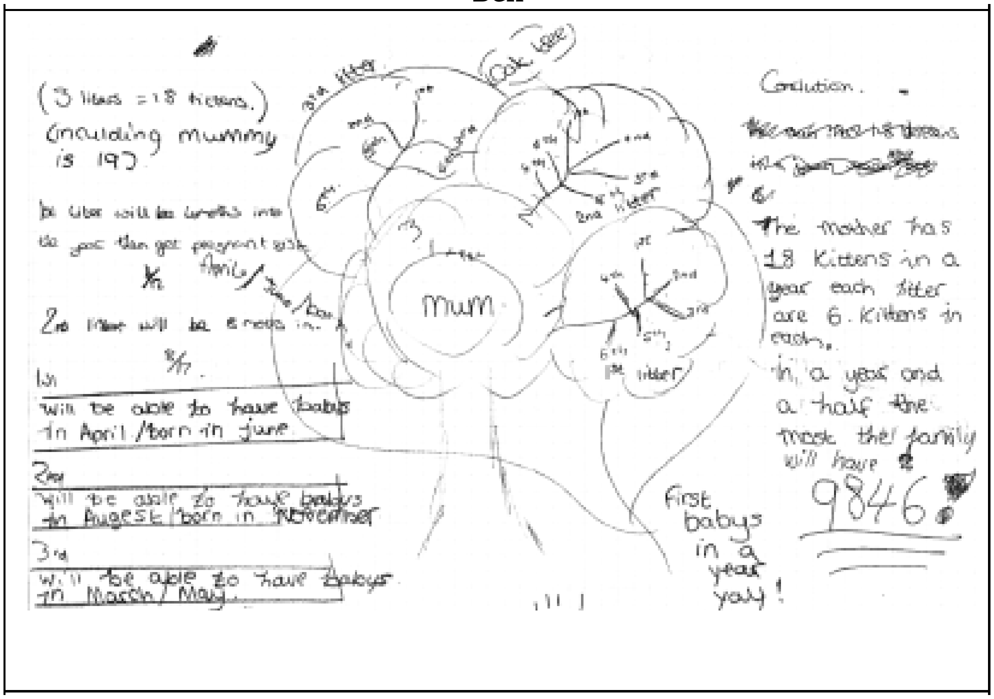
Question 5. What suggestions would you make to Alice?

Wayne



Question 6. What comments would you make to Wayne?

Ben



Question 7. What comments would you make to Ben?

Question 8. Your task now is to produce a solution that is better than your individual solutions. Explain your reasoning.

Handout 5 **Name:** ______________________________
Week # 3: 1/26/2021
Math 1150: Winter 2021
Instructor: Andrade

Introduction:
Last class we got a taste of how mathematics can be used to make predictions. In this exercise, we will discuss two sample solutions to the problem from last time and how assumptions alter our predictions. Next we will formalize the concept of a mathematical model. For the next few class periods, we will be investigating some well-known mathematical models and the tools needed to operate with these models.

Having Kittens

Recall the poster we discussed last class. Today we will discuss two different approaches to the solution and how the differences in assumptions can yield dramatically different results.

Solution 1

Generation of kittens

Time in months

	0	1	2	2	2	2	3	3	3	4	Cum.total
0	1	6									7
1											7
2											7
3											7
4		6									13
5											13
6			36								49
7											49
8		6									55
9											55
10			36	36							127
11											127
12		6					216				349
13											349
14			36	36	36						457
15											457
16		6					216	216	216		1111
17											1111
18			36	36	36	36				1296	2551

Solution 2

Generation of kittens

Time in months

	0	1	2	2	2	3	3	Cum. total
0	1							1
1								1
2		3						4
3								4
4								4
5								4
6		3						7
7								7
8			9					16
9								16
10		3						19
11								19
12			9	9				37
13								37
14		3				27		67
15								67
16			9	9	9			94
17								94
18		3				27	27	151

Question 1. Having worked on this problem yourself, take a moment to compare these solutions to your own. How were the difficulties you encountered handled in these solutions? Write your reflections here.

Question 2. From the information shown in the charts, what assumptions have been made in Solution 1 and Solution 2?

Question 3. What are the major differences in the assumptions of Solution 1 and Solution 2. Discuss how these assumptions impacted the final count?

Question 4. What changes would you make to your own solution now? Explain why you would make or not make changes.

Mathematical Models

Mathematics is most often used as a means of describing or predicting the behavior of some real-life system. The mathematical description of a system or a phenomenon is called a **mathematical model**.

Question 5. Can you think of an example of a mathematical model being used in your current field of study? What questions are being answered? What assumptions are being made?

Question 6. Draw a diagram that illustrates the mathematical modeling process.

Question 7. List and describe the differences between the three types of mathematical models we discussed in class.

What type of model would best describe the scenario of counting cat descendants?

Handout 6 **Name:** ____________________
Week # 3: 1/28/2021
Math 1150: Winter 2021
Instructor: Andrade

Introduction:
There are several types of functions that can be used to model relationships observed in the real world. Today we will discuss how some familiar functions are used to model real life phenomena.

Linear Functions

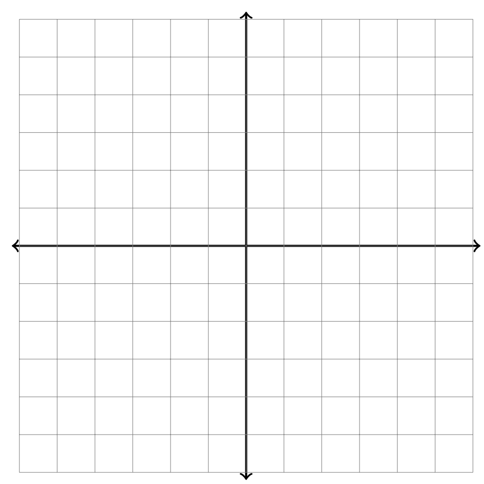
When we say that y is a **linear function** of x, we mean that the graph of the function is a line. Recall that we can represent a line by

$$y = mx + b,$$

where m is the slope and b is the y-intercept. On the other hand, if we knew that (x_1, y_1) and (x_2, y_2) are two points on a line, then the slope of that line is given by

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}.$$

Question 1. Find the slope of the line passing through $(-1, 3)$ and $(1, -1)$. Then sketch the graph.



A characteristic feature of linear functions is that they grow at a constant rate. In other words, we should consider linear functions as descriptive tools for situations where the quantity of interest is changing at a constant rate.

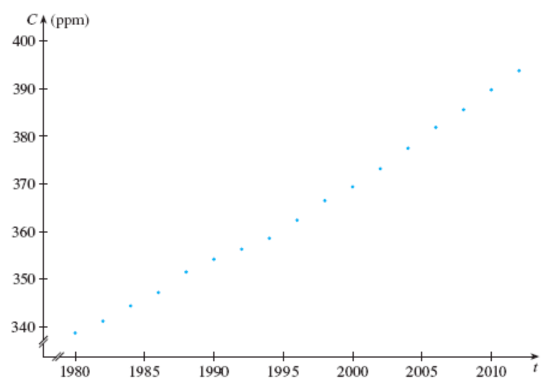
Question 2. Do you think a linear function would be a suitable model to describe the number of descendants one female cat can have in 18 months? Why or why not?

Question 3. As dry air moves upward, it expands and cools. If the ground temperature is $20°$ C and the temperature at a height of 1 km is $10°$ C, express the temperature T (in $°$ C) as a function of height h (in kilometers), assuming that a linear model is appropriate.

a.) Use your model to predict the temperature at a height of 2.5 km.

Question 4. The given table lists the average carbon dioxide level in the atmosphere, measured in parts per million at Mauna Loa Observatory from 1980 to 2012. Use the data to find a model for the carbon dioxide level.

Year	CO_2 Level (in ppm)	Year	CO_2 Level (in ppm)
1980	338.7	1998	366.5
1982	341.2	2000	369.4
1984	344.4	2002	373.2
1986	347.2	2004	377.5
1988	351.5	2006	381.9
1990	354.2	2008	385.6
1992	356.3	2010	389.9
1994	358.6	2012	393.8
1996	362.4		



a.) Do you believe that a linear model appropriate for this data? Why or why not?

b.) Use the data to find a model for the carbon dioxide level.

c.) Using a linear regression calculator, compute the regression line.

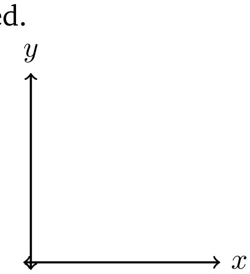
d.) Use the model in part (c) to predict when the level of CO_2 will exceed 420 parts per million.

e.) Between the models found in part (b) and part (c), which would you expect to be more accurate? Explain why.

Choosing the right function for the job.

Linear functions certainly cannot accurately describe every situation we encounter. However, you probably have some familiarity with the toolbox of functions we will need.

Question 5. Consider a function $h(t)$ that models the height of a person given their age in t years. Draw a sketch of what the function might look like.



Handout 7 **Name:** ____________________________
Week # 5: 2/11/2021
Math 1150: Winter 2021
Instructor: Andrade

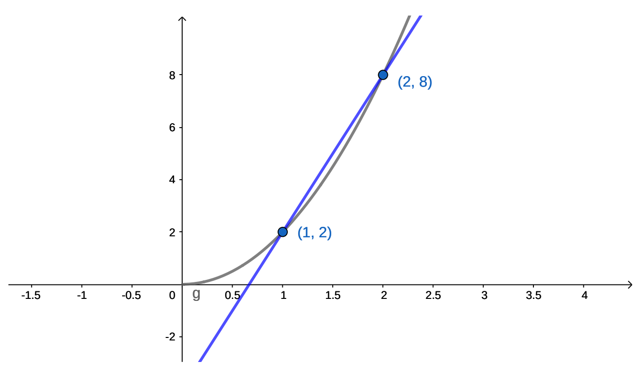
Introduction:
As was discussed in the readings, Newton developed calculus in part to answer questions he had about motion and rates of change. On Tuesday, we discussed some of the mathematical handicaps Newton experienced and the techniques developed to tackle this problems. In this handout, you will apply those techniques to make predictions about the rate of change.

Rates of Change and Tangent Lines

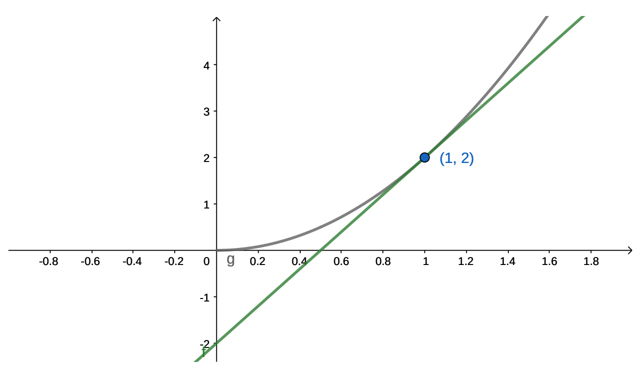
Recall that the tangent line to a curve is one that touches the curve at a point and captures the "slope" of the curve (at that given point). A secant line is a straight line passing through two points on the given curve.

Question 1. Suppose the graph g below describes the distance a free-falling object travels after t seconds of flight. In class on Tuesday, we talked about how the secant and tangent lines hold important information about the motion of the object. Answer the following questions.

a.) What information does the secant line (shown in the figure) between the two points when $t = 1$ and $t = 2$ tell us about the speed of the object during this period?



b) What information does the tangent line (shown in the figure) when $t = 1$ tell us about the speed of the object?



c.) In your own words, describe how the secant lines are used to obtain the slope of the tangent line. You may want to include a graph.

Following Newton's Approach

Question 2. Suppose that a ball is dropped from the upper observation deck of the CN tower in Toronto, 450 m above the ground. Galileo's law states that the distance fallen after t seconds can be represented as

$$s(t) = 4.9t^2,$$

where $s(t)$ is given in meters.

a.) Compute the average speed the ball travels between $t = 5$ and $t = 6$. You may use a calculator, but include your calculation steps below. Your final answer should include units.

b.) Complete the following table. You may use a computing device or calculator and do not need to include your calculations for this part.

Interval	Average Speed
$t = 5$ to $t = 6$	
$t = 5$ to $t = 5.1$	
$t = 5$ to $t = 5.05$	
$t = 5$ to $t = 5.01$	
$t = 5$ to $t = 5.001$	
$t = 5$ to $t = 5.0001$	

c.) Use the previous part to determine the instantaneous speed (or velocity) when $t = 5$? Your answer should include units.

d.) Based on the graph of the function $s(t)$, how do you think the instantaneous speed of the ball when $t = 9$ compares to speed of the ball when $t = 5$? You may want to graph the function using a graphing utility. Do not base your answer on intuition, but instead use the graph to support your answer.

Using the rate of change to predict the future.

Question 3. Consider a population of bacteria in a homogeneous nutrient medium. Suppose that by sampling the population at certain intervals it is determined that the population doubles every hour.

a.) If the initial population is 10 and t is measured in hours, how many bacteria will be present after 1 hour? After 2 hours? After 3 hours?

b.) Use the previous part, to devise a model (or function) $P(t)$ that describes the number of bacteria present after t hours. In other words, find a pattern using part (a) so that you can mathematically describe the number of bacteria after t hours.

c.) Using your model from part (b), predict how many bacteria will be present after 24 hours.

d.) In your own words, what does the rate of change of $P(t)$ represent? Is the rate of change constant? Explain your reasoning.

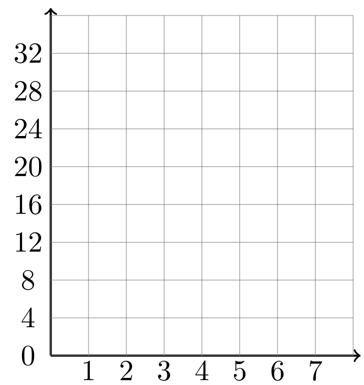
Handout 8 **Name:** ______________________________
Week # 6: 2/18/2021
Math 1150: Winter 2021
Instructor: Andrade

Introduction:
On the last handout, we saw a scenario where bacteria in a petri dish double every hour. This example illustrates that not all functions can be represented by linear functions. Today we will investigate the rate of change of exponential functions and determine when these functions might help in describing real-life scenarios.

Question 1. Use the function $y = 2^x$ to answer the following questions.

a.) Complete the following table for the y-values of the function $y = 2^x$.Then using the points in your table, graph the function.

x	$y = 2^x$
0	$2^0 = 1$
1	
2	
3	
4	
5	



b.) Next compute the slopes between each of the points above. What do you notice?

Points	Slope
$x = 0$ to $x = 1$	$\frac{2-1}{1-0} = 1$
$x = 1$ to $x = 2$	
$x = 2$ to $x = 3$	
$x = 3$ to $x = 4$	
$x = 4$ to $x = 5$	

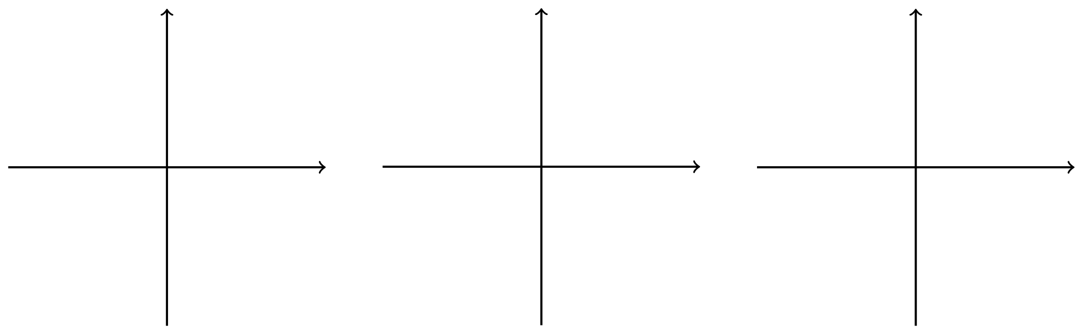
Exponential functions take the form $y = a \cdot b^x$. For our purposes we will assume both a and b are positive numbers.

Question 2. What does the parameter a represent? Specifically what does a tell you about the graph of the function $y = a \cdot b^x$? What might a represent in a real life scenario?

Question 3. From the previous exercises, what do you know about the rate of change of an exponential function? For example, is it constant? Can you describe the rate of change of $y = ab^x$ in terms of a and b?

Question 4. Use a graphing utility to investigate each of the following cases. Draw a sample graph for each

$0 < b < 1$ $\qquad$ $b = 1$ $\qquad$ $1 < b$



Question 5. Several scenarios are given below. Identify each as best modeled by a linear function, an exponential function, or neither. Explain your choice. You may want to suggest an equation to support your argument.

- A gym membership costs \$ 50 to open then \$ 25 per month thereafter.

- The number of kids at a theme park triples every hour.

- In the year 2000, an apple cost 10 cents. Today, it costs 50 cents.

- A rocket accelerates from the surface of the moon.

- The number of descendants a single cat can have in 18 months.

- A rock is dropped from a cliff.
- Every year, a sandwich shop doubled its profits.
- The more money a car manufacturer spends on advertising, the more cars they sell.
- The more rainfall there is in the morning, the more students are late to school.
- A plane descends 2 meters vertically for every 5 meters of horizontal movement.
- In the springtime, the number of ants in a colony quadruples every week.
- In the fall, the number of leaves on a particular tree is halved every week.